2017 （　　　　　）

EXAMINATION QUESTIONS

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COLLEGE OF TECHNOLOGY STUDENTS

MATHEMATICS

60

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

Nationality		No.			Marks	
	(Please print full name, underlining family name)					
Name						

MATHEMATICS

1   Answer the following questions and write your answers in the boxes provided.

1) Solve the equation $x^3 + 2x^2 - x - 2 = 0$.

$$x =$$

2) Solve the equation $\sin 2x = \sqrt{3}\cos x \quad (0 \le x \le 2\pi)$.

$$x =$$

3) Solve the equation $2\log_3(x+2) = \log_3(10-x)$.

$$x =$$

4) Solve the inequality $3^{2x+1} - 2 \cdot 3^x - 1 < 0$.

5) Solve the inequality $(\log_3 x)(\log_{27} x) - \log_9 x + \dfrac{1}{6} < 0.$

6) Let $O(0,0)$, $A(1,1)$ and $B(x,-1)$. Find the value of x such that
$\angle AOB = \dfrac{\pi}{3}.$

$x =$

7) Find the radius of the sphere $x^2 + y^2 + z^2 = 2x + 6y - 1.$

8) Find the tangent line to the curve $y = \dfrac{1}{2}x^2 + 3$ $(x \geq 0)$ which goes through the point $(0,\ 0).$

$y =$

9) Calculate $\displaystyle\sum_{k=1}^{100} \frac{1}{k(k+1)}$.

10) Calculate $\displaystyle\lim_{x\to\frac{1}{2}} \frac{16x^4 - 1}{2x - 1}$.

11) Let $f(x) = \cos^3 x$. Calculate $f'\left(\dfrac{\pi}{6}\right)$.

$$f'\left(\frac{\pi}{6}\right) = $$

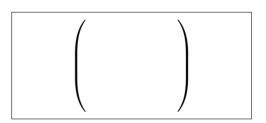12) Calculate $\displaystyle\int_1^{e^2} x\log x\, dx$.

2　Let $A = \begin{pmatrix} 1 & 2 \\ a & b \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ satisfy $A^2 - A = 2I$. Answer the following questions and write your answers in the boxes provided.

　1) Find a and b.

$$a = \qquad b =$$

　2) Find A^3.

$$A^3 = \begin{pmatrix} & \\ & \end{pmatrix}$$

　3) Find $A^6 - A^5 + A^4 - A^3 + A^2 - A$.

$$\begin{pmatrix} & \\ & \end{pmatrix}$$

3 Let $f(x) = xe^{-x^2}$. Answer the following questions and write your answers in the boxes provided.

1) Find the derivative $f'(x)$.

$$f'(x) =$$

2) Find the indefinite integral $\displaystyle\int f(x)\,dx$.

3) Calculate $\displaystyle\lim_{a\to\infty}\int_0^a f(x)\,dx$.